Lenses (E7)

Objectives

- Measure the focal lengths of converging and diverging lenses.
- Examine properties of partially covered thin lenses.
- Measure the index of refraction of a cylindrical lens.

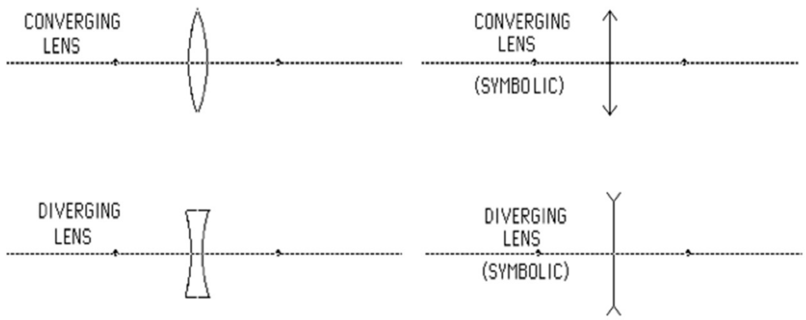
Theory

Index of refraction *n* of a material is the ratio of speed of light *c* in a vacuum to the speed of light in the material *v*.

$$\text{index of refraction } = n = \frac{\text{speed of light in a vacuum}}{\text{speed of light in a medium}} = \frac{c}{v} \qquad (1)$$

Thin Lenses

Lenses posses the ability to create images by refracting light in certain ways. In general, there are two main types of lenses: converging (convex) and diverging (concave) lenses.



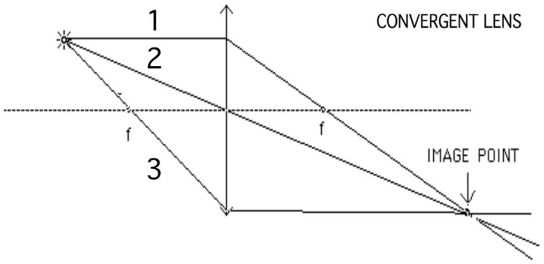
A converging lens tends to focus light rays while a diverging lens causes the light to spread (or to diverge).

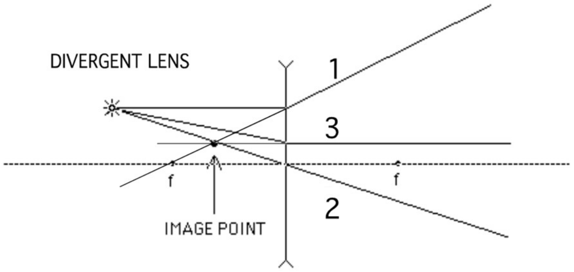
The image position for lenses can be determined both geometrically and algebraically. For the geometric solution, there are three principal rays as shown below.

RAY #1: The first ray is drawn parallel to the optic axis. After being refracted by the lens, this ray then passes through (or appears to come from) one of the focal points. This ray is called the *parallel ray*.

RAY #2: The second ray is drawn through the center of the lens. This ray continues in a straight line. This ray is called the *center ray*.

RAY #3: The third ray is drawn through the focal point f and emerges from the lens parallel to the optic axis. This ray is called the *focal ray*.





The image distance may also be found algebraically.

The focal length *f* is equal to the distance from the lens to the focal point. The focal length *f* is positive for converging lenses and negative for diverging lenses. Once the focal length is determined, the image distance can be easily determined from the following **thin-lens equation**:

$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f} \quad \left\{ \begin{array}{ll} \text{where:} & s_o = \text{object distance from the lens ("+" to the left)} \\ & s_i = \text{image distance from the lens ("+" to the right, real image)} \\ & \qquad \text{and "-" to the left, virtual image)} \\ & f = \text{focal length ("+" for converging lens, "-" for the diverging lens)} \end{array} \right\} \tag{2}$$

The **magnification** *m* is given by the following equation:

$$m=\frac{\text{Image height}}{\text{Object height}}=-\frac{s_i}{s_o} \tag{3a}$$

If $m > 0$, then the image is upright with respect to the object.

If $m < 0$, then the image is inverted with respect to the object.

The magnification m_θ of a magnifying glass is given by the following formula:

$$m_\theta=\frac{s_N}{f} \qquad \text{where the near-point distance } s_N = 0.25\text{ m} = 25\text{ cm} \tag{3b}$$

The **near-point distance** s_N is the nearest distance to get a sharp image of an object for a human with perfect eyes. The commonly approved value is $s_N = 0.25$ m.

Sign conventions for lenses.

Quantity	When Positive ("+")	When Negative ("-")
Object distance s_o	The object is to the left of the lens (real object)	The object is to the right of the lens
Image distance s_i	The image is to the right of the lens (real image)	The image is to the left of the lens (virtual image)
Focal length f	Converging lens or concave mirror	Diverging lens or convex mirror
Magnification m	Upright image	Inverted image

Refraction (Snell's Law)

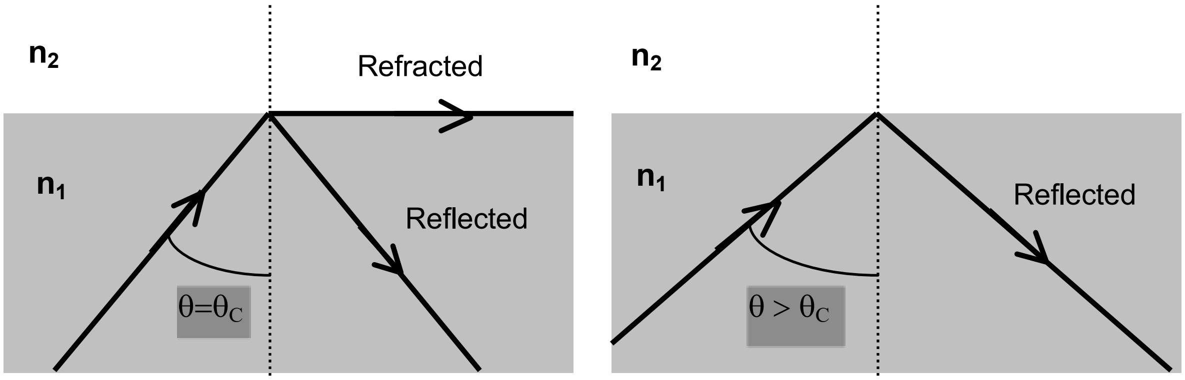
Snell's law relates the refraction angle θ_2 to the incident angle θ_1 and the indexes of refraction of the incident medium n_1 and the refracted medium n_2.

$$n_2 sin\theta_2 = n_1 sin\theta_1$$

Total Internal Reflection

When light passes from a medium of larger index of refraction n_1 into one of smaller index of refraction n_2, the refracted ray is bent *away* from the normal. If the incident ray is at the **critical angle** θ_C, the angle of refraction is 90˚.

When the angle of incidence exceeds the critical angle, all the incident light is reflected back into the medium from which it came, a phenomenon known as **total internal reflection**.



Assuming the medium of lesser index of refraction is air with $n_2 = 1.00$ we get the following equation (using Snell's law) for the **critical angle** θ_c:

$$\sin\theta_c = \frac{n_2}{n_1} \quad \Rightarrow \quad \sin\theta_c = \frac{1}{n_1} = \frac{1}{n} \quad \text{where } n_2 = 1.00 \quad \text{and} \quad n_1 = n \qquad (4)$$

Procedure:

Activity 1: Focal Length of Converging and Diverging Lenses.

In this Activity, we will measure the focal length of converging and diverging lenses.

1.1. ***Converging (convex) lens***.

If the object distance s_o and the image distance s_i are measured directly, then the focal length f can be calculated using the thin-lens equation. Put the light source to the track. Make sure that the crossed arrow object faces the converging lens. See Figure 1a below.

1.2. Place the object at distance s_{oc} = 15 cm, 20 cm, 25 cm, 30 cm, 40 cm from the converging lens, and locate the sharp image of the crossed arrow object on the screen. Here "s_{oc}" indicates the object distance for the converging lens. Our object here is the highlighted pattern on the screen of the light source.

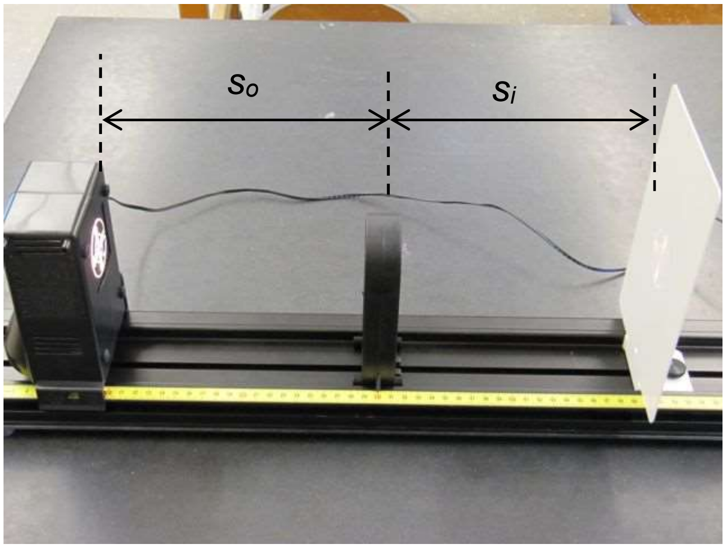
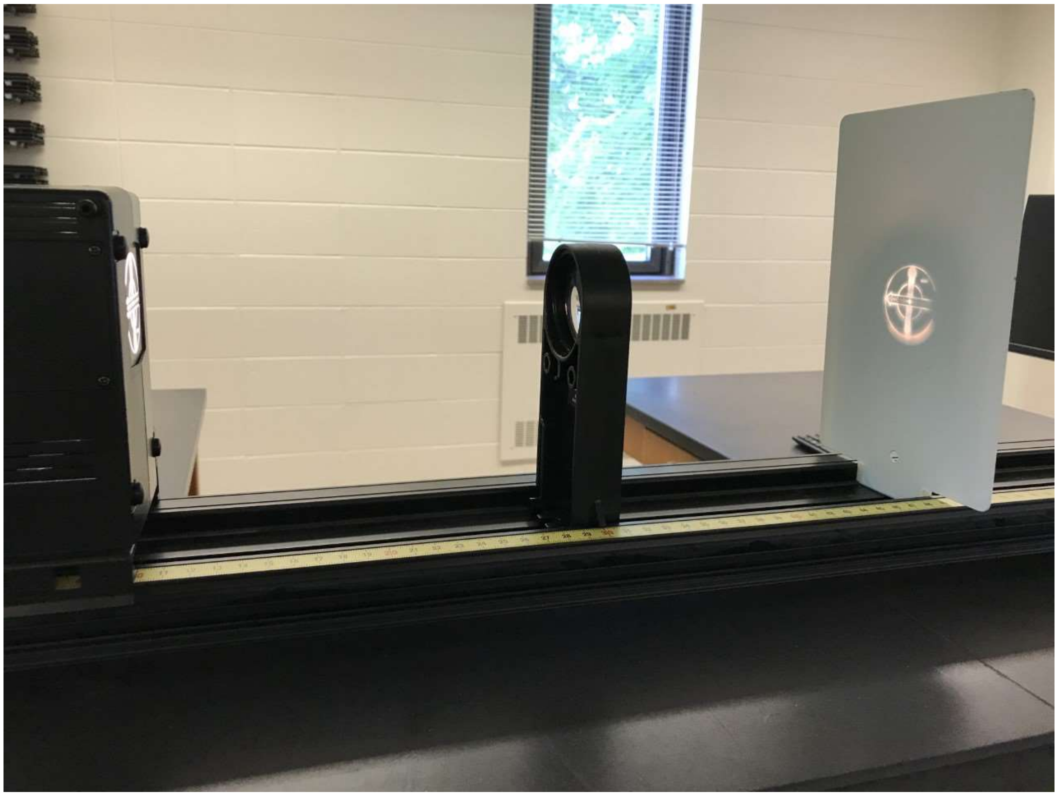


Fig. 1a.



1.3. When you measure the distance to the lens use the center of the lens as the position of the lens. Measure s_{oc} and s_{ic} carefully and calculate the focal length of the converging lens f_c from these measurements. Find the average focal length value.

1.4. Try to reduce the object distance to be smaller than the focal length ($s_{oc} < f_c$). Are you able to find a real image on the screen?

1.5. Each converging lens could be used as a magnifying glass. Using the average focal length and the usual value of the near-point distance $s_N = 25$ cm, calculate the angular magnification for this converging lens when used as the magnifying glass. Observe the enlarged image of the tip of your pen.

1.6. ***Diverging (concave) lens.***

A diverging lens cannot create real image on the screen. Therefore, to measure the focal length of a diverging lens, we will use two lenses simultaneously: the converging lens that you just measured and the diverging lens available on the table. Remember, that the focal length of a diverging lens is negative. Here is the experimental setup (*the distance between lenses and the screen on the drawing is not accurate or proportional to the real setup*).

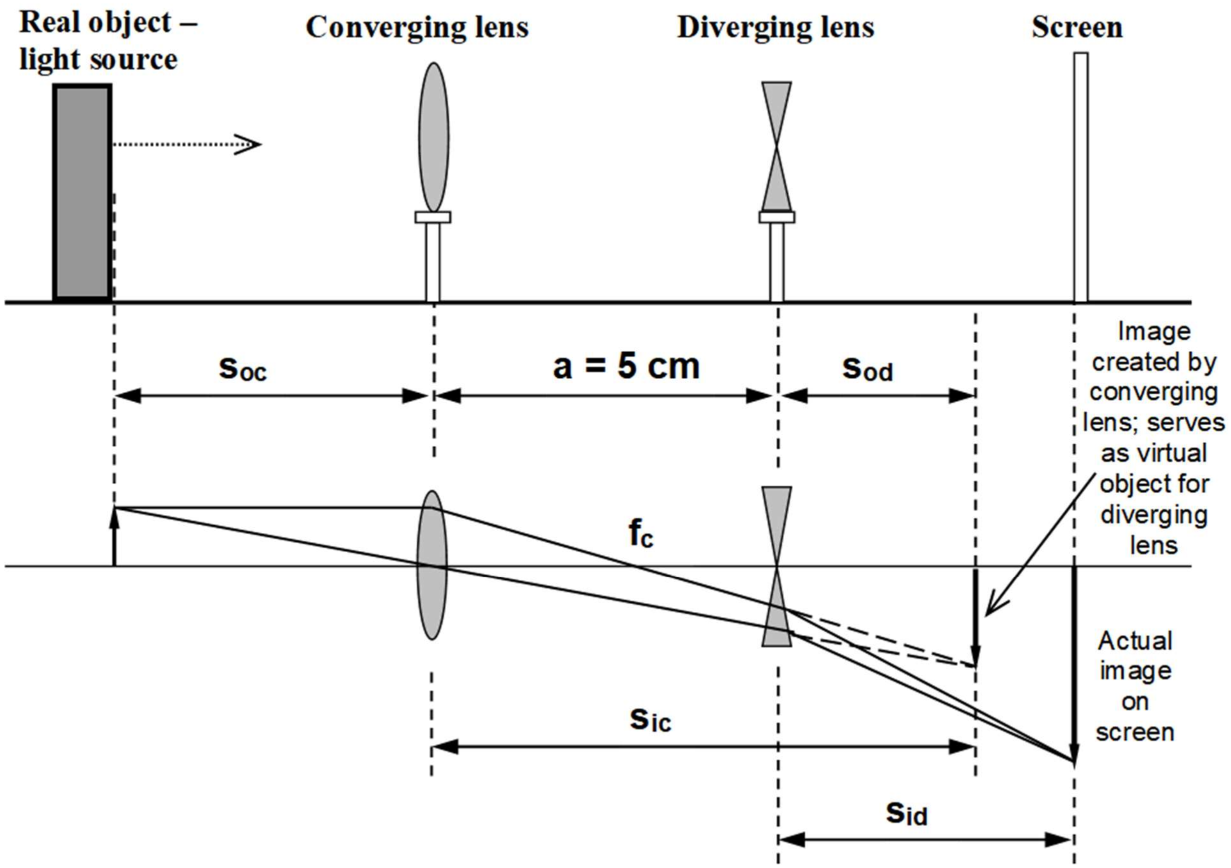
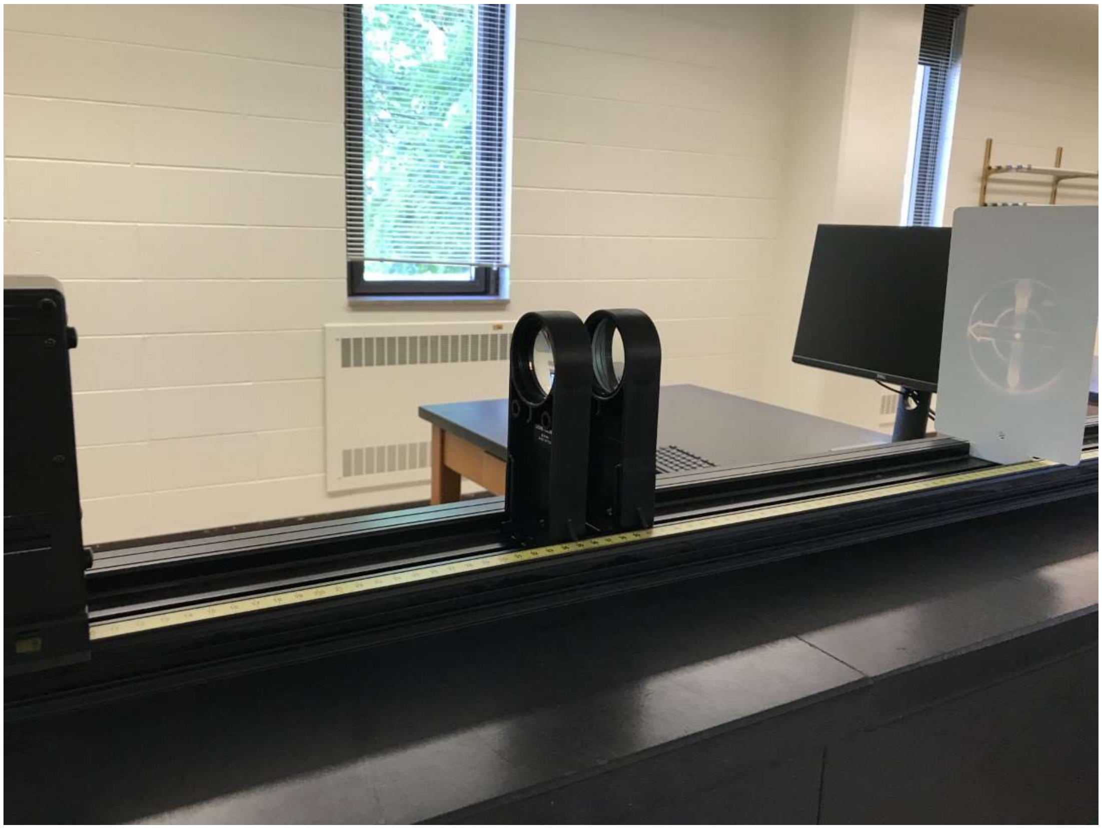

Real object – light source
Converging lens
Diverging lens
Screen
s_{oc}
a = 5 cm
s_{od}
Image created by converging lens; serves as virtual object for diverging lens
f_c
Actual image on screen
s_{ic}
s_{id}

Fig. 1b.



The image created by the converging lens (at the distance s_{ic} from the converging lens) is also the object for the diverging lens. The object for the diverging lens is on the right side of the lens, so the object distance for the diverging lens s_{od} must be negative. If the separation between lenses is equal to a, then the distance between the diverging lens and the object for the diverging lens s_{od} is equal to:

$$s_{od} = -(s_{ic} - a) = a - s_{ic} = 5\ cm - s_{ic} \qquad (s_{od} \text{ is negative}) \tag{5}$$

Here "s_{od}" indicates the object distance for the diverging lens. Now, we will apply the thin-lens equation to calculate the focal length of the diverging lens f_d.

$$\frac{1}{f_d} = \frac{1}{s_{od}} + \frac{1}{s_{id}} = \frac{s_{od}+s_{id}}{s_{od}\times s_{id}} \quad \Rightarrow \quad f_d = \frac{s_{od}\times s_{id}}{s_{od}+s_{id}} \tag{6}$$

1.7. Place the object at distance s_{oc} = 25 cm, 30 cm, 40 cm from the converging lens. Add the diverging lens a = 5 cm to the right of the converging lens and locate the sharp image of the crossed arrow object on the screen.

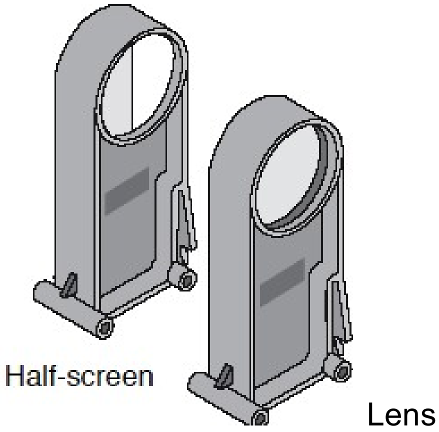
1.8. Copy s_{ic} values for the converging lens alone from the data table (Activity 1.3.) Use the lens separation a = 5 cm to calculate the object distance for the diverging lens $s_{od} = a - s_{ic}$ = 5 cm -s_{ic} (Eq. (5)). Measure the distance from the diverging lens to the position of the image - s_{id}.

1.9. Next, use Eq. (6) to calculate the focal length of the diverging lens f_d. Keep in mind that the focal length of a diverging lens should be negative! Repeat calculations for the other distances between the object and the converging lens s_{oc}. Calculate the average focal length of the diverging lens.

Activity 2: Partially Covered Lenses

2.1. Use converging lens and prepare the same type of setup as for Activity 1.2. Make sure that the crossed arrow object (part of the light source) faces the converging lens. Place the lens at the distance equal to 1.5 times the average focal length (1.5 * f_c from *Activity* 1) from the object. Move the screen to get a sharp image of the crossed arrow.

2.2. Add the provided “half-screen” accessory next to the lens and cover one half of the lens (left or right).



What has happened to the image? With the lens still partially covered, do you see a part of the image or a complete image? If you see a complete image, then please explain how it is possible.

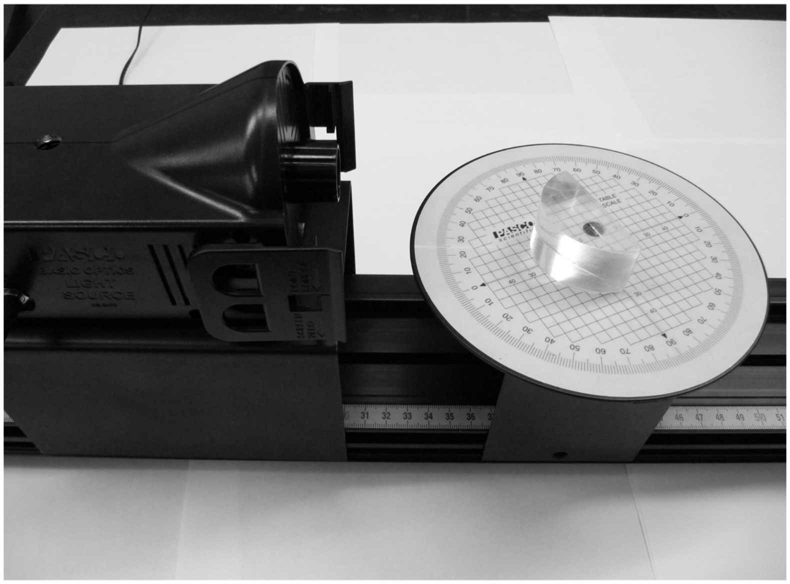
2.3. Remove the lens cover and the lens.

Activity 3: Index of Refraction of a Cylindrical Lens.

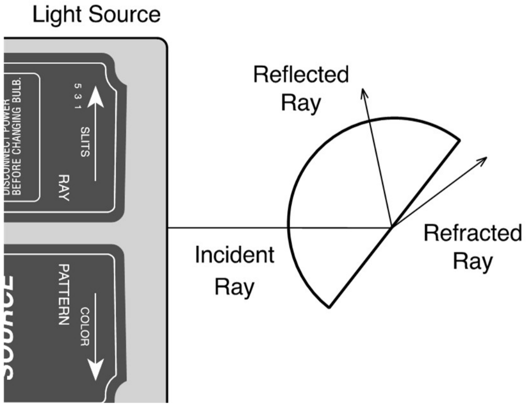
The purpose of this activity is to find index of refraction of a cylindrical lens and to measure the critical angle for the total internal reflection.

3.1. Put the ray table on the base with the "RAY TABLE mm SCALE" side facing up. Turn the ray table so the 0 (zero) degree line points to the light source.

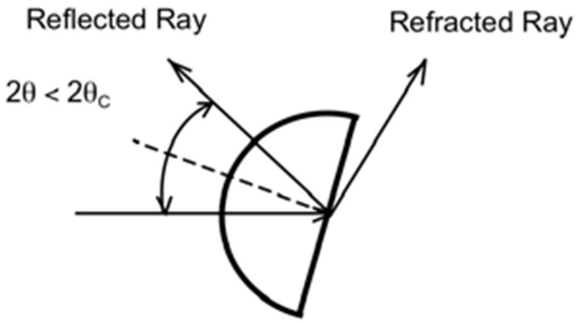
3.2. Put the light source on its bracket so that the multiple slits are facing the Ray Table (you will need to rotate the light source). Position the light source so it is about two centimeters from the edge of the ray table. Adjust the slit mask on the front of the light source so the **light source projects only one (central) ray of light** across the middle of the top surface of the ray table.

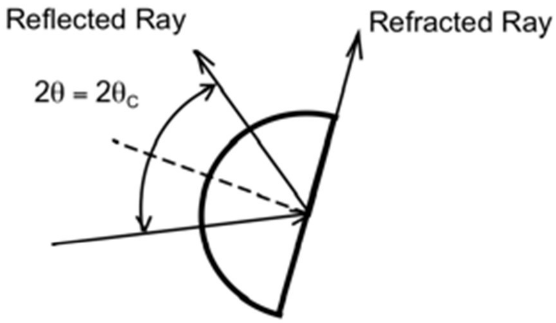


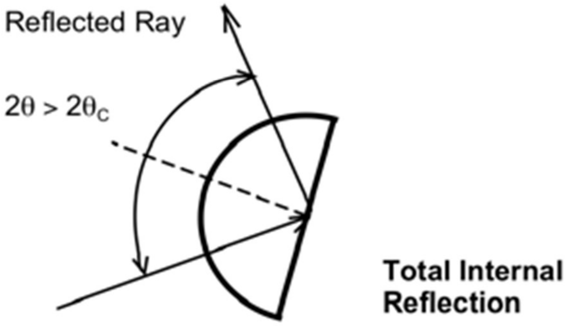
3.3. Put the acrylic, **cylindrical lens** on the ray table so the curved surface of the lens faces the light source, and the edge of the lens is on the 90º (ninety) degree line with the lens exactly centered along the 90° degree line. The light ray should shine on the 0º (zero) degree line.

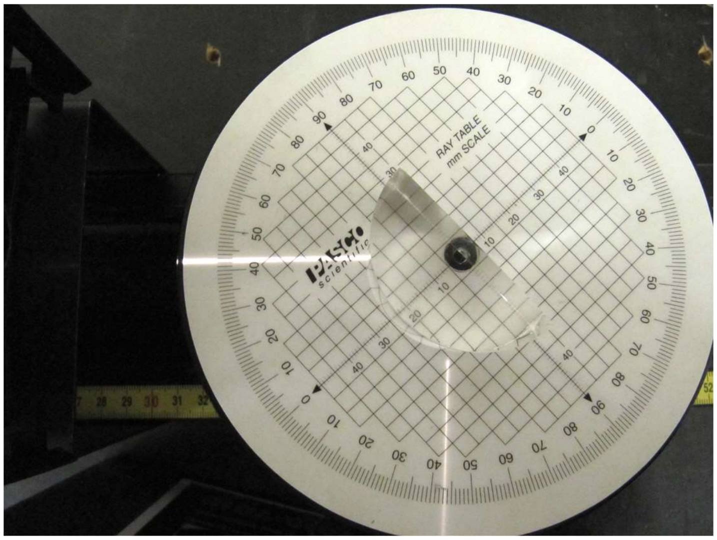


3.4. Identify incident ray, reflected ray and refracted ray. Rotate the ray table until the refracted light ray emerging from the cylindrical lens just barely disappears. Hold a piece of white paper next to the edge of the ray table so you can see the light ray. Just as it disappears, the ray separates into colors. Stop at that point and measure the total angle between the incident and reflected rays. Note that this total angle is twice the **critical angle** because the backside of lens is flat, and the angle of reflection equals the angle of incidence.









3.5. When finished please **disconnect the light source from the power and remove it** from the track.

Make sure to complete the following tasks:

You must submit the answers to the prelaboratory questions online. (3.5 points)

1. Your completed Data Sheets. (6.5 points)
2. Return the completed lab report to your lab TA.